MERSENNE'S PRIME NUMBERS([1])

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Abstract

The authors present several lemmas and theorems on Mersenne's prime numbers. Some theorems are new. Of particular interest is the proof of primality tests on the Mersenne composite and the demonstration that the "New Mersenne Conjecture" is true.

So the authors say:

M3	prime	MM3	prime!
M5	prime	MM5	NOT prime
M7	prime	MM7	prime!
M13	prime	MM13	NOT prime
M17	prime	MM17	NOT prime
M19	prime	MM19	NOT prime
M31	prime	MM31	prime!
M61	prime	MM61	NOT prime
M89	prime	MM89	NOT prime
M107	prime	MM107	prime!
M127	prime	MM127	prime!
M521	prime	MM521	NOT prime
M607	prime	MM607	prime!

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[1] *The paper is on the database of CNR Solar* http://150.146.3.132/

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Lemma 1
A Mersenne's prime number Mp is always of form 4k+3.

Dem.
If Mp=2^p-1=4k+1 then:

4k+1=2^p-1 -> 4k+2=2^p-> 2(2k+1)=2^p -> 2k+1 = 2^(p-1)

It's absurd: odd=even!

so

if Mp=2^p-1=4k+3 then:
4k+3=2^p-1 -> 4k+4=2^p-> 4(k+1)=2^p -> k+1 = 2^(p-2) -> k=2^(p-2)-1
It is possible.

Euler showed:
Theorem: If k>1 and p=4k+3 is prime, then 2p+1 is prime if and only if 2p = 1 (mod 2p+1).

so
Lemma 2
if p=4k+3 or p=4k-1 a prime number and 2p+1 is a prime number then the Mersenne number Mp= 2^p-1 is composite.

Lemma 3
Let be p = 4k-1 a prime number, with k>0 and k = 0 mod 3 then Mp=2^p-1 isn't a prime number".

Dem.
It follows from Lemma 2

Lemma 4 equivalent to Lemma 3
If p = 4k-1 is a prime number, if are true two conditions:
a. (p+1) mod 4 = 0
b. ((p+1)/4) mod 3 = 0
then Mp is composite

Examples:
p=11=4*3-1 k=3*1 Mp=2^11-1 not prime
p=23=4*6-1 k=3*2 Mp=2^23-1 not prime
p=47=4*12-1 k=3*4 Mp=2^47-1 not prime

Lemma 5
If W =(4k+5)/3 is an odd with k>1 and W>3, if W is a Wagstaff's prime number then k = Mp is a Mersenne's prime Number.

Dem.
If W is a Wagstaff's prime number then w=(2^p+1)/3 an p is a prime number.
Mp= 2^p-1 so W=(Mp+2)/3 but from Lemma 1 Mp=4k+3 then W=(4k+5)/3

If p is a prime number and W a Wagstaff prime number then k integer is a Mersenne's prime number if 4k+5/3 is integer.

Theorem MMp
MMp is prime if Mp is prime and exists a k>=0 such that (Mp*3-5)/4 must be integer.

Dem.
Let be W=(2^p+1)/3 a Wagstaff's prime number, Mp=2^p-1 a Mersenne's prime number.

From Lemma 1 is Mp=4k+3
From Lemma 5 a Wagstaff's prime number is W=(Mp+2)/3=(4k+5)/3 or k=Mp=(3W-5)/4

Primality test for MMp
(Mp*3-5)/4 is a primality test for MMp.

Examples:

M3	prime	MM3	prime!
M5	prime	MM5	NOT prime
M7	prime	MM7	prime!
M13	prime	MM13	NOT prime
M17	prime	MM17	NOT prime
M19	prime	MM19	NOT prime
M31	prime	MM31	prime!
M61	prime	MM61	NOT prime
M89	prime	MM89	NOT prime
M107	prime	MM107	prime!
M127	prime	MM127	prime!
M521	prime	MM521	NOT prime
M607	prime	MM607	prime!

Theorem New Mersenne Conjecture
For each p prime number, if are true two conditions:
a. p=2^k ± 1 or p = 4^k ± 3.
b. (2^p + 1) / 3 is a prime number (Wagstaff)
then Mp=2^p-1 is a Mersenne's prime number.

Dem.
The demonstration follows from lemmas previous.
If Lemma 3 is true (p = 4k-1 with k>1 and k=0 mod3), Mp isn't a prime, so to avoid that we can choose p=2^k± 1

For example:
2^2+1=4*1+1=5
2^4+1=4*4+1=17
2^8+1=4*64+1=257

The prime numbers of form 4^k± 3 are of type 4k±3 and in this mode we can find Mersenne's prime numbers.

For example:
4^1+3=4*1+3=7
4^2+3=4*4+3=19
4^3+3=67